NEARLY DEGENERATE NEUTRINO MASSES AND NEARLY DECOUPLED NEUTRINO OSCILLATIONS [a]

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We introduce a simple flavor symmetry breaking scheme, in which charged lepton masses have a strong hierarchy and neutrino masses are almost degenerate. It is possible to obtain a natural suppression of lepton flavor mixing between the 1st and 3rd families as well as the approximate decoupling of atmospheric and solar neutrino oscillations with nearly bi-maximal mixing factors. The similarity and difference between lepton and quark flavor mixing schemes are briefly discussed.

In the standard model neutrinos are assumed to be the massless Weyl particles. But most extentions of the standard model (such as the grand unified theories of quarks and leptons) allow the existence of massive neutrinos, although the masses of three active neutrinos ν_e, ν_μ and ν_τ could be much smaller than those of their charged counterparts. Whether the smallness of the masses of three neutrinos are attributed to the neutrality of their electric charges or to the Majorana feature of their fields, remains an open question.

The recent observation of the atmospheric and solar neutrino anomalies, particularly that in the Super-Kamiokande experiment, has provided strong evidence that neutrinos are massive and lepton flavors are mixed [1]. Analyses of the atmospheric neutrino deficit in the framework of two-flavor neutrino oscillations yield the following mass-squared difference and mixing factor:

$$\Delta m^2_{\rm atm} \;\sim\; 10^{-3}\ {\rm eV}^2 \; , \qquad\qquad \sin^2 2\theta_{\rm atm} \;>\; 0.8 \; . \tag{1}$$

In addition, the hypothesis that solar ν_e neutrinos change to another active species through long-wavelength vacuum oscillations with the parameters

$$\Delta m^2_{\rm sun} \;\sim\; 10^{-10}\ {\rm eV}^2 \; , \qquad\qquad \sin^2 2\theta_{\rm sun} \;\approx\; 1 \; , \tag{2}$$

can provide a consistent explanantion of all existing solar neutrino data [2]. In the framework of three-flavor neutrino oscillations, the significant hierarchy between $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sun}$ together with the no observation of $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillation in the CHOOZ experiment implies that the ν_3-component in ν_e is rather small (even negligible) and the atmospheric and solar neutrino oscillations approximately decouple [3]. If this picture is true, then the solar and atmospheric

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neutrino deficits should mainly attributed to the corresponding $\nu_e \rightarrow \nu_\mu$ and $\nu_\mu \rightarrow \nu_\tau$ transitions. In this case,

$$\begin{aligned} \Delta m^2_{\rm sun} &= \Delta m^2_{21} \equiv \left| m^2_2 - m^2_1 \right| \ , \\ \Delta m^2_{\rm atm} &= \Delta m^2_{32} \equiv \left| m^2_3 - m^2_2 \right| \ , \end{aligned} \tag{3}$$

and $\Delta m^2_{31} \approx \Delta m^2_{32}$. Nevertheless, the hierarchy of Δm^2_{21} and Δm^2_{32} (or Δm^2_{31}) does not give information about the absolute values or the relative magnitude of three neutrino masses. For example, either the strongly hierarchical neutrino mass spectrum ($m_1 \ll m_2 \ll m_3$) or the nearly degenerate one ($m_1 \approx m_2 \approx m_3$) is allowed to reproduce the "observed" gap between Δm^2_{21} and Δm^2_{32}.

In this talk we pay attention only to the mass degeneracy of active neutrinos, which might be good candidates for the hot dark matter of the universe. We introduce a simple flavor symmetry breaking scheme for charged lepton and neutrino mass matrices, so as to generate two nearly bi-maximal flavor mixing angles and to interpret the approximate decoupling of solar and atmospheric neutrino oscillations. Within the scope of this discussion, we do not take the LSND evidence for neutrino oscillations and the matter-enhanced mechanism for solar neutrino oscillations into account.

Let us start with the symmetry limits of the charged lepton and neutrino mass matrices. In a specific basis of flavor space, in which charged leptons have the exact flavor democracy and neutrino masses are fully degenerate, the mass matrices can be written as [4,5]

$$M^{(0)}_l = \frac{c_l}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \ , \qquad M^{(0)}_\nu = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \ , \tag{4}$$

where $c_l = m_\tau$ and $c_\nu = m_0$ measure the corresponding mass scales. If the three neutrinos are of the Majorana type and CP symmetry is conserved, $M^{(0)}_\nu$ could take a more general form $M^{(0)}_\nu P_\nu$, where $P_\nu = {\rm Diag}\{\eta_1, \eta_2, \eta_3\}$ with $\eta_i = \pm 1$ denoting the CP parities. For simplicity we neglect the effect of P_ν, which is only relevant to the neutrinoless double beta decay, in the subsequent discussions. Clearly $M^{(0)}_\nu$ exhibits an S(3) symmetry, while $M^{(0)}_l$ an $S(3)_{\rm L} \times S(3)_{\rm R}$ symmetry. In these limits $m_e = m_\mu = 0$, $m_1 = m_2 = m_3 = m_0$, and no flavor mixing is present.

A simple diagonal breaking of the flavor democracy for $M^{(0)}_l$ and the mass degeneracy for $M^{(0)}_\nu$ may lead to instructive results for neutrino oscillations [4]. Let us proceed with two different symmetry-breaking steps.

(i) Small perturbations to the (3,3) elements of $M^{(0)}_l$ and $M^{(0)}_\nu$ are respec-

tively introduced [6]:

$$\Delta M_l^{(1)} \;=\; \frac{c_l}{3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_l \end{pmatrix} \;, \qquad \Delta M_\nu^{(1)} \;=\; c_\nu \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_\nu \end{pmatrix} \;. \tag{5}$$

In this case the charged lepton mass matrix $M_l^{(1)} = M_l^{(0)} + \Delta M_l^{(1)}$ remains symmetric under an $S(2)_{\rm L} \times S(2)_{\rm R}$ transformation, and the neutrino mass matrix $M_\nu^{(1)} = M_\nu^{(0)} + \Delta M_\nu^{(0)}$ has an $S(2)$ symmetry. The muon becomes massive ($m_\mu \approx 2|\varepsilon_l| m_\tau/9$), and the mass eigenvalue m_3 is no more degenerate with m_1 and m_2 (i.e., $|m_3 - m_0| = m_0|\varepsilon_\nu|$). After the diagonalization of $M_l^{(1)}$ and $M_\nu^{(1)}$, one finds that the 2nd and 3rd lepton families have a definite flavor mixing angle θ. We obtain $\tan\theta = -\sqrt{2}$ if the small correction of $O(m_\mu/m_\tau)$ is neglected. Then neutrino oscillations at the atmospheric scale may arise in $\nu_\mu \rightarrow \nu_\tau$ transitions with $\Delta m^2_{32} = \Delta m^2_{31} \approx 2m_0|\varepsilon_\nu|$. The corresponding mixing factor $\sin^2 2\theta \approx 8/9$ is in good agreement with current data [1].

(ii) Small perturbations, which have the identical magnitude but the opposite signs [4], are introduced to the (2,2) and (1,1) elements of $M_l^{(1)}$ or $M_\nu^{(1)}$:

$$\Delta M_l^{(2)} = \frac{c_l}{3}\begin{pmatrix} -\delta_l & 0 & 0 \\ 0 & \delta_l & 0 \\ 0 & 0 & 0 \end{pmatrix} \;, \qquad \Delta M_\nu^{(2)} = c_\nu \begin{pmatrix} -\delta_\nu & 0 & 0 \\ 0 & \delta_\nu & 0 \\ 0 & 0 & 0 \end{pmatrix} \;. \tag{6}$$

We obtaine $m_e \approx |\delta_l|^2 m_\tau^2/(27 m_\mu)$ and $m_1 = m_0(1-\delta_\nu)$, $m_2 = m_0(1+\delta_\nu)$. The diagonalization of $M_l^{(2)} = M_l^{(1)} + \Delta M_l^{(2)}$ and $M_\nu^{(2)} = M_\nu^{(1)} + \Delta M_\nu^{(2)}$ leads to a full 3×3 flavor mixing matrix, which links neutrino mass eigenstates (ν_1, ν_2, ν_3) to neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ in the following manner:

$$V \;=\; \frac{1}{\sqrt{6}}\begin{pmatrix} \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & -2 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} \;\pm\; \xi_V \sqrt{\frac{m_e}{m_\mu}} \;\mp\; \zeta_V \;\frac{m_\mu}{m_\tau} \;, \tag{7}$$

where

$$\xi_V = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 1 & -2 \\ -\sqrt{3} & \sqrt{3} & 0 \\ 0 & 0 & 0 \end{pmatrix} \;, \quad \zeta_V = \frac{1}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \\ -1 & -1 & 2 \end{pmatrix} \;. \tag{8}$$

Some comments on this result are in order.

- This mixing pattern, after neglecting small corrections from the charged lepton masses, is similar to that of the pseudoscalar mesons π^0, η and η'

in QCD [6,7]:

$$
\begin{aligned}
|\pi^0\rangle &= \frac{1}{\sqrt{2}}\left(|\bar{u}u\rangle - |\bar{d}d\rangle\right) \,, \\
|\eta\rangle &= \frac{1}{\sqrt{6}}\left(|\bar{u}u\rangle + |\bar{d}d\rangle - 2|\bar{s}s\rangle\right) \,, \\
|\eta'\rangle &= \frac{1}{\sqrt{3}}\left(|\bar{u}u\rangle + |\bar{d}d\rangle + |\bar{s}s\rangle\right) \,.
\end{aligned}
\tag{9}
$$

One is invited to speculate whether such an analogy could be taken as a hint towards an underlying symmetry responsible for lepton mass generation [8].

- The (1,3) element of V is naturally suppressed in the symmetry breaking scheme outlined above. A similar feature appears in the 3×3 quark flavor mixing matrix, i.e., $|V_{ub}|$ is the smallest among the nine quark mixing elements. Indeed the smallness of V_{e3} provides a necessary condition for the decoupling of solar and atmospheric neutrino oscillations, even though neutrino masses are nearly degenerate. The effect of small but nonvanishing V_{e3} can manifest itself in the long-baseline $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\tau$ transitions, as we shall see below.

- The flavor mixings between the 1st and 2nd lepton families and between the 2nd and 3rd lepton families are nearly maximal. This property, together with the natural smallness of V_{e3}, allows a satisfactory interpretation of the observed large mixing in atmospheric and solar neutrino oscillations. We obtain

$$
\sin^2 2\theta_{\rm sun} \;=\; 1 \,, \qquad \sin^2 2\theta_{\rm atm} \;=\; \frac{8}{9}\left(1 \mp \frac{m_\mu}{m_\tau}\right) \tag{10}
$$

  to a high degree of accuracy. At present both solutions for $\sin^2 2\theta_{\rm atm}$, i.e., $\sin^2 2\theta_{\rm atm} = 0.84$ or 0.94, are allowed by data.

Let us make a brief but useful comparison between the lepton and quark flavor mixing schemes. For simplicity we make use of the following parametrization [9]:

$$
V \;=\; \begin{pmatrix} s_x s_y c + c_x c_y e^{-{\rm i}\phi} & s_x c_y c - c_x s_y e^{-{\rm i}\phi} & s_x s \\ c_x s_y c - s_x c_y e^{-{\rm i}\phi} & c_x c_y c + s_x s_y e^{-{\rm i}\phi} & c_x s \\ -s_y s & -c_y s & c \end{pmatrix} \,. \tag{11}
$$

For leptons we take the subscripts $x = l$ and $y = \nu$, while for quarks $x = {\rm u}$ and $y = {\rm d}$. Therefore the rotation angle θ_l (or θ_ν) mainly describes the mixing

between e and μ leptons (or between ν_e and ν_μ neutrinos), and the rotation angle $\theta_{\rm u}$ (or $\theta_{\rm d}$) primarily describes the mixing between u and c quarks (or between d and s quarks). The rotation angle θ is a combined effect arising from the mixing between the 2nd and 3rd families, for either quarks or leptons. The phase parameter ϕ signals CP violation in flavor mixing (for neutrinos of the Majorana type, two additional CP-violating phases may enter but they are irrelevant for neutrino oscillations). Comparing Eqs. (7) and (11) we immediately arrive at (up to a sign ambiguity of θ_l)

$$\tan\theta_l \;=\; \sqrt{\frac{m_e}{m_\mu}} \;, \qquad \tan\theta_\nu \;=\; 1 \;. \tag{12}$$

In contrast, a variety of quark mass matrices predict [10,11]

$$\tan\theta_u \;=\; \sqrt{\frac{m_u}{m_c}} \;, \qquad \tan\theta_d \;=\; \sqrt{\frac{m_d}{m_s}} \;. \tag{13}$$

As one can see, the large mixing angle θ_ν is attributed to the near degeneracy of neutrino masses in our flavor symmetry breaking scheme.

Finally we consider the effect of nonvanishing θ_l for $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\tau$ transition probabilities in long-baseline (LB) neutrino experiments, in which the oscillations associated with the mass-squared difference Δm^2_{21} can safely be neglected. We obtain [5]

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e)_{\rm LB} &= \frac{16}{9}\frac{m_e}{m_\mu}\sin^2\left(1.27\,\frac{\Delta m^2_{32}L}{|\mathbf{P}|}\right) \;, \\ P(\nu_e \rightarrow \nu_\tau)_{\rm LB} &= \frac{8}{9}\frac{m_e}{m_\mu}\sin^2\left(1.27\,\frac{\Delta m^2_{32}L}{|\mathbf{P}|}\right) \;. \end{aligned} \tag{14}$$

The mixing factors in these two processes are 0.8% and 0.4%, respectively. The former might be within the sensitivity region of MINOS.

More data from the Super-Kamiokande and other neutrino experiments could finally clarify whether the solar neutrino deficit is attributed to the long-wavelength vacuum oscillation. They will provide stringent tests of the model discussed here.

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